IIT-JAM
Subject- Sequence

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Lecture in Mathematics

May 2020

1. Which of the following is/are correct?

a. $nlog(1 + \frac{1}{n+1}) \longrightarrow 1$ as $n \longrightarrow \infty$

b. $(n+1)log(1 + \frac{1}{n+1}) \longrightarrow 1$ as $n \longrightarrow \infty$

c. $n^2 log(1 + \frac{1}{n}) \longrightarrow 1$ as $n \longrightarrow \infty$

d. $nlog(1 + \frac{1}{n^2}) \longrightarrow 1$ as $n \longrightarrow \infty$

2. If $< x_n >$ and $< y_n >$ are sequences of real numbers, which of the following is are true?

a. $\limsup(x_n + y_n) \leq \limsup x_n + \limsup y_n$

b. $\limsup(x_n + y_n) \geq \limsup x_n + \limsup y_n$

c. $\liminf(x_n + y_n) \leq \liminf x_n + \liminf y_n$

d. $\liminf(x_n + y_n) \geq \liminf x_n + \liminf y_n$

3. Let $x_n = n^{\frac{1}{n}}$ and $(n!)^{\frac{1}{n}}$, $n \geq 1$ be two sequence of real numbers. Then

a. x_n converge, but not y_n does not converge.

b. y_n converge, but x_n does not converge.

c. Both x_n and y_n converge.

d. Neither x_n nor y_n converge.

4. Let a_n be a sequence of positive number

a. $\lim_{n\to\infty}(a_n)^{\frac{1}{n}} \leq 1 \Longrightarrow \lim_{n\to\infty}(\frac{a_{n+1}}{a_n}) \leq 1$

b. $\lim_{n\to\infty}(\frac{a_{n+1}}{a_n}) \leq 1 \Longrightarrow \lim_{n\to\infty}(a_n)^{\frac{1}{n}} \leq 1$

c. $\lim_{n\to\infty}(\frac{a_{n+1}}{a_n}) \leq 1 \iff \lim_{n\to\infty}(a_n)^{\frac{1}{n}} \leq 1$

d. $\lim_{n\to\infty}(\frac{a_{n+1}}{a_n})$ may exist but $\lim_{n\to\infty}(a_n)^{\frac{1}{n}}$ may not exist.

5. Let $a_n = (-1)^n sin(\frac{n\pi}{2}), n = 1, 2, 3, ...$ Then

a. $\liminf a_n = -1$ and $\limsup a_n = 1$

b. $\liminf a_n = 0$ and $\limsup a_n = 1$

c. $\liminf a_n = -1$ and $\limsup a_n = 0$

d. $\liminf a_n = 1$ and $\limsup a_n = -1$

6.

$$\sup_n\{(\sqrt{n+1} = \sqrt{n})\}$$

and

$$\limsup_n(\sqrt{n+1} - \sqrt{n})$$

are

a. $\sqrt{2} + 1$ and 0 respectively

b. $\frac{1}{\sqrt{2}+1}$ and 0 respectively

c. Both equal to 0

d. Both equal to $\frac{1}{\sqrt{2}+1}$

7. Let a_n and b_n be sequence of real numbers defined as $a_1 = 1$ and for $n \geq 1, a_{n+1} = a_n + (-1)^n 2^{-n}$, $b_n = \frac{2a_{n+1}-a_n}{a_n}$. Then

a. a_n converges to zero and b_n is a cauchy sequence.

b. a_n converges to a non-zero number and b_n is a cauchy sequence.

c. a_n converges to zero and b_n is not a convergent sequence.

d. a_n converges to a non-zero number and b_n is not a convergent sequence.

8. Let a_n be a sequence of distinct real numbers, which has no convergent subsequence. Then $\lim_{n\to\infty} |a_n|$ is

a. 0

b. ∞

c. 1

d. does not exist

9.Consider the sequence $< x_n >_{n\geq 1}$ defined recursively as $x_1 = 1$, $x_n = sup\{x \in [0, x_{n-1}) : sin(\frac{1}{x}) = 0\}, x \geq 2$ Then, $\limsup_{n\to\infty}$ equals

a. 0

b. $-\infty$

c. 1

d. ∞

10. $x_{n+1} = \frac{-3}{4}x_n, x_0 = 1$. The sequence x_n

a. diverges

b. x_n is monotonically increasing and converges to 0

c. x_n is monotonically decreasing and converges to 0

d. None of the above

11. The sequence $\sqrt{7}, \sqrt{7+\sqrt{7}}, \sqrt{7+\sqrt{7+\sqrt{7}}}, ...$ converges to

a. $\frac{1+\sqrt{33}}{2}$

b. $\frac{1+\sqrt{32}}{2}$

c. $\frac{1+\sqrt{30}}{2}$

d. $\frac{1+\sqrt{29}}{2}$

12. Let a_1, a_2 be positive real numbers and let $a_{n+1} = \frac{1}{2}(a_n + a_{n-1})$ for $n \geq 2$. Then the sequences a_{2n} and a_{2n-1}

a. are not bounded

b. both diverge to ∞

c. both converge to the same limit

d. both converges to different limits

13. Consider the following statements

1. A bounded sequence in $\mathbb{R}$ need not be convergent.
2. A bounded sequence in $\mathbb{R}$ need not have a convergent subsequence.
3. A bounded sequence in $\mathbb{R}$ need not have a constant subsequence.

a. All three statements are true.

b. None of these statements are true.

c. 1 and 2 are true but 3 is false.

d. only 1 is true.

14. Let $f(x) = \sqrt{x} sin\frac{1}{x}$ for $x > 0$. Set $x_n = f(\frac{2}{(2n+1)\pi})$. Then sequence x_n

a. Is a non-converging oscillatory sequence

b. diverges to ∞

c. converges to $\sqrt{\pi}$

d. converges to 0

15. Consider the sequence a_n of real numbers where $a_1 > 1$ and $a_{n+1} = 2 - \frac{2}{a_n}, n \geq 1$. then the sequence a_n is

a. Bounded but not monotone.

b. not bounded but monotone

c. Both bounded and monotone.

d. Neither bounded nor monotone.

16. Let x_n be an unbounded sequence of non-zero real numbers. Then

a. x_n must have a convergent subsequence

b. x_n cannot have a convergent subsequence

c. $\frac{1}{x_n}$ must have a convergent subsequence

d. $\frac{1}{x_n}$ cannot have a convergent subsequence

17. Consider the sequence x_n denoted by $x_n = \frac{[nx]}{n}$ for $x \in \mathbb{R}$, where [.] denotes the integer part. Then x_n

a. Converges to x

b. converges but not to x

c. does not converge

d. Oscillates

18. Suppose a_n, b_n are convergent sequence of real number, such that $a_n > 0$ and $b_n > 0$ for all n. Suppose $lim_{n \longrightarrow \infty} a_n = a$ and $\lim_{n \longrightarrow \infty} b_n = b$. Let $c_n = \frac{a_n}{b_n}$ Then

a. c_n converges if $b > 0$

b. c_n converges only if $a = 0$

c. c_n converges only if $b > 0$

d. $\limsup c_n = \infty$ if $b = 0$

19.Let $< x_n >$ and $< y_n >$ be two sequence of real numbers. Then

a. $x_n \longrightarrow \infty$ and $y_n \longrightarrow \infty \Longrightarrow < x_n y_n > \longrightarrow \infty$

b. $x_n \longrightarrow \infty$ and $y_n \longrightarrow 0 \Longrightarrow < x_n y_n > \longrightarrow 0$

c. $x_n \longrightarrow \infty$ and y_n is bounded $\Longrightarrow < x_n y_n >$ is bounded

d. x_n unbounded and y_n unbounded $\Longrightarrow < x_n y_n >$ is unbounded.

20. Let $a_n \longrightarrow a$. Let for every positive integer k, A_k be the set of all positive integers n such that $|a_n - a| < \frac{1}{k}$. Then

a. A_k is finite for all k

b. A_k is finite for some k

c. Every A_k contains all but finitely many positive integers

d. Some A_k contains all positive integers.

21. Let $x_n = \frac{n}{2^n}, n > 1$ be areal sequence, $\forall n \in \mathbb{N}$. Then which of the following is/are **TRUE**

a. $< x_n >$ is bounded.

b. $< x_n >$ is strictly increasing

c. $< x_n >$ is strictly decreasing

d. Limit of the sequence does not exist.

22. The sequence $< x_n >$ defined recursively by $x_1 = \frac{3}{2}$, $x_n = \sqrt{3x_{n-1} - 2}$, for $n \geq 2$, Then which of the following is/are **NOT TRUE**

a. $< x_n >$ is cauchy sequence.

b. $< x_n >$ is monotonic increasing

c. $< x_n >$ is monotonic decreasing

d. $< x_n >$ is convergent to 2

23. The sequence $< x_n >$ defined recursively as follows $x_1 = 5$, $x_{n+1} = (x_n - 5)^2$, for $n \geq 1$, Then which of the following is/are **TRUE**

a. The sequence $< x_n >$ is strictly increasing and bounded above.

b. $< x_n >$ is strictly increasing but unbounded above.

c. $< x_n >$ is bounded below

d. $< x_n >$ is convergent to 2

24. The sequence x_n satisfy $0 < x_n < 1, x_n(1 - x_{n+1}) > \frac{1}{9}$, for $n \in \mathbb{N}$. Then which of the following is/are **TRUE**

a. The sequence $< x_n >$ is decreasing and bounded below by 2.

b. The limit point of the sequence $< x_n >$ is 2.

c. $< x_n >$ is convergent to 2

d. All of the above

25. The sequence defined by $a_1 = 0, a_{n+1} = \sqrt{6 + a_n}$, for $n \geq 1$. then which of the following is/are **TRUE**

a. The sequence $< a_n >$ is convergent but not cauchy.

b. The sequence $< a_n >$ is convergent to 3.

c. Every subsequence of $< a_n >$ is convergent.

d. Non of these.

26. Let x_n be a sequence of real numbers which is does not converges to $x \in \mathbb{R}$. Then which of the following is/are **TRUE**

a. There exist an $\epsilon > 0$ such that, for any $k \in \mathbb{N}, \exists n_k \in \mathbb{N}$ such that $|x_{n_k} - x| \geq \epsilon, \forall n_k \geq k$

b. for every $\epsilon > 0, \exists$ a natural number $k \in \mathbb{N}$ such that $|x_{n_k} - x| \geq \epsilon, \forall n_k \geq k$.

c. There exist an $\epsilon_0 > 0$ and a subsequence $X' =< x_{n_k} >$ of $< x_n >$ such that $|x_{n_k} - x| \geq \epsilon_0, \forall k \in \mathbb{N}$

d. $< x_n >$ has two convergent subsequence $< x_{n_k} >$ and x_{r_k}, whose limits are not equal.

27. Let $y_1 = \sqrt{p}$, where $p > 0$ and $y_{n+1} = \sqrt{p + y_n}$, for $n \in \mathbb{N}$. Then

a. $< y_n >$ is bounded sequence.

b. $< y_n >$ is monotonic increasing

c. $< y_n >$ is bounded above by $\frac{1}{2}(1 + \sqrt{1 + 4p})$.

d. $< y_n >$ is convergent

28. The sequence defined by $a_1 = 0, a_2 = \frac{1}{2}, a_{n+1} = \frac{1}{3}(1 + a_n + a_{n_1}^3)$, for $n > 1$. Then

a. $< a_n >$ is convergent sequence.

b. limit of the sequence is 1

c. The limit of the sequence is $\frac{-1+\sqrt{5}}{2}$

d. All of the above.

29. Let s_n be a sequence of positive real numbers satisfying

$$2s_{n+1} = s_n^2 + \frac{3}{4}, n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is (are) **TRUE**?

a. $< s_n >$ is monotonically decreasing.

b. $< s_n >$ is monotonically increasing.

c. $\lim_{n\to\infty} s_n = \alpha$.

d. $\lim_{n\to\infty} s_n = \beta$.

30. The sequence $a_n = \sqrt{A^2n^2 + n + 1} - n$ is

a. Convergent for all real values of A to x

b. Divergent for all real values A

c. Convergent for exactly one real value of A

d. Convergent for exactly two real values of A